Problem 3

MAX da + db + dc + dd + de + df + dg + dh + di + dj + dk + dl + dm

ST

da = 0

db - da < 2

dc - da < 3

dd - da < 8

dh - da < 9

da - db < 4

dc - db < 5

de - db < 7

df - db < 4

dd - dc < 10

db - dc < 5

dg - dc < 9

di - dc < 11

df - dc < 4

da - dd < 8

dg - dd < 2

dj - dd < 5

df - dd < 1

dh - de < 5

dc - de < 4

di - de < 10

di - df < 2

dg - df < 2

dd - dg < 2

dj - dg < 8

dk - dg < 12

di - dh < 5

dk - dh < 10

da - di < 20

dk - di < 6

dj - di < 2

dm - di < 12

di - dj < 2

dk - dj < 4

dl - dj < 5

dh - dk < 10

dm - dk < 10

dm - dl < 2

da > 0

db > 0

dc > 0

dd > 0

de > 0

df > 0

dg > 0

dh > 0

di > 0

dj > 0

dk > 0

dl > 0

dm > 0

END

1. What are the lengths of the shortest paths from vertex a to all other vertices.

VARIABLE VALUE REDUCED COST

DA 0.000000 0.000000

DB 2.000000 0.000000

DC 3.000000 0.000000

DD 8.000000 0.000000

DE 9.000000 0.000000

DF 6.000000 0.000000

DG 8.000000 0.000000

DH 9.000000 0.000000

DI 8.000000 0.000000

DJ 10.000000 0.000000

DK 14.000000 0.000000

DL 15.000000 0.000000

DM 17.000000 0.000000

1. If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a).

The distance value dz for that vertex is unbounded because there are no constraints imposed on its value by edge weights, so you can increase the maximum forever by just increasing dz.

1. What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?

MAX da + db + dc + dd + de + df + dg + dh + di + dj + dk + dl + dm

ST

dm = 0

da - db < 2

da - dc < 3

da - dd < 8

da - dh < 9

db - da < 4

db - dc < 5

db - de < 7

db - df < 4

dc - dd < 10

dc - db < 5

dc - dg < 9

dc - di < 11

dc - df < 4

dd - da < 8

dd - dg < 2

dd - dj < 5

dd - df < 1

de - dh < 5

de - dc < 4

de - di < 10

df - di < 2

df - dg < 2

dg - dd < 2

dg - dj < 8

dg - dk < 12

dh - di < 5

dh - dk < 10

di - da < 20

di - dk < 6

di - dj < 2

di - dm < 12

dj - di < 2

dj - dk < 4

dj - dl < 5

dk - dh < 10

dk - dm < 10

dl - dm < 2

da > 0

db > 0

dc > 0

dd > 0

de > 0

df > 0

dg > 0

dh > 0

di > 0

dj > 0

dk > 0

dl > 0

END

VARIABLE VALUE REDUCED COST

DA 17.000000 0.000000

DB 15.000000 0.000000

DC 15.000000 0.000000

DD 12.000000 0.000000

DE 19.000000 0.000000

DF 11.000000 0.000000

DG 14.000000 0.000000

DH 14.000000 0.000000

DI 9.000000 0.000000

DJ 7.000000 0.000000

DK 10.000000 0.000000

DL 2.000000 0.000000

DM 0.000000 0.000000

For this version, instead of starting with a source vertex s and finding the shortest paths to all other vertices in the graph, we start from target vertex t and find all vertices that point to t, and work our way outward from there. The LINDO program is almost identical, except the target is set with the “= 0” constraint and we swap the operands of the subtraction operator in all of the edge constraints to reverse the direction.

1. Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all x,y ∈ V)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).

We can use the same linear program as part c) and set the target to vertex i. This gives us the shortest path from every reachable vertex u to vertex i. Any unreachable vertex is unbounded. Then we run the same linear program as part a) and set the source to vertex i. This gives us the shortest path from vertex i to every reachable vertex v. Once again, any unreachable vertex is unbounded. Finally, we simply add the values together for every permutation (u, i, v) for every u, v ∈ V to get the shortest path from u to v via i.

VARIABLE SP TO DI SP FROM DI

DA 8.000000 20.000000

DB 6.000000 22.000000

DC 6.000000 23.000000

DD 3.000000 28.000000

DE 10.000000 29.000000

DF 2.000000 26.000000

DG 5.000000 28.000000

DH 5.000000 16.000000

DI 0.000000 0.000000

DJ 2.000000 2.000000

DK 15.000000 6.000000

DL UNBOUNDED 7.000000

DM UNBOUNDED 9.000000

Shortest Path Via i (δ(u,i,v))

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| v  u | a | b | c | d | e | f | g | h | i | j | k | l | m |
| a | 28 | 30 | 31 | 36 | 37 | 34 | 36 | 24 | 8 | 10 | 14 | 15 | 17 |
| b | 26 | 28 | 29 | 34 | 35 | 32 | 34 | 22 | 6 | 8 | 12 | 13 | 15 |
| c | 26 | 28 | 29 | 34 | 35 | 32 | 34 | 22 | 6 | 8 | 12 | 13 | 15 |
| d | 23 | 25 | 26 | 31 | 32 | 29 | 31 | 19 | 3 | 5 | 9 | 10 | 12 |
| e | 30 | 32 | 33 | 38 | 39 | 36 | 38 | 26 | 10 | 12 | 16 | 17 | 19 |
| f | 22 | 24 | 25 | 30 | 31 | 28 | 30 | 18 | 2 | 4 | 8 | 9 | 11 |
| g | 25 | 27 | 28 | 33 | 34 | 31 | 33 | 21 | 5 | 7 | 11 | 12 | 14 |
| h | 25 | 27 | 28 | 33 | 34 | 31 | 33 | 21 | 5 | 7 | 11 | 12 | 14 |
| i | 20 | 22 | 23 | 28 | 29 | 26 | 28 | 16 | 0 | 2 | 6 | 7 | 9 |
| j | 22 | 24 | 25 | 30 | 31 | 28 | 30 | 18 | 2 | 4 | 8 | 9 | 11 |
| k | 35 | 37 | 38 | 43 | 44 | 41 | 43 | 31 | 15 | 17 | 21 | 22 | 24 |
| l | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| m | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |